

3D Trigonometry Questions By Topic:



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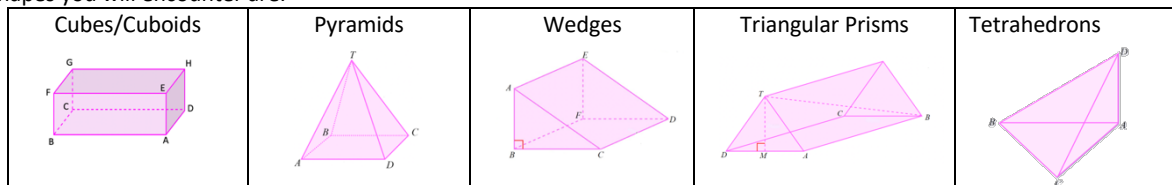
This topic is essentially knowing which right angled triangle(s) to form and then using Pythagoras and SOHCAHTOA on the triangles formed. In other words, we look to locate the right-angled triangles in 2D within these 3D shapes and then use:

- Pythagoras (if only sides are involved)
- SOHCAHTOA (if angles and sides are involved)



So, if you're good at Pythagoras and SOHCAHTOA then you'll find this topic easy. If you're not comfortable with Pythagoras and SOHCAHTOA, make sure you go back and learn these topics first, otherwise you'll struggle.

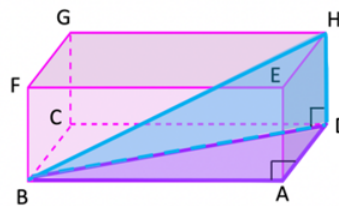
The five shapes you will encounter are:



The most common shapes encountered are cuboids, pyramids, prisms and tetrahedrons.

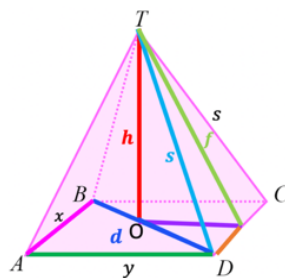
Cuboids

We commonly look at the following 2 right-angled triangles (purple and blue). We usually use Pythagoras or SOHCAHTOA on triangle ABD and Pythagoras or SOHCAHTOA on triangle BDH

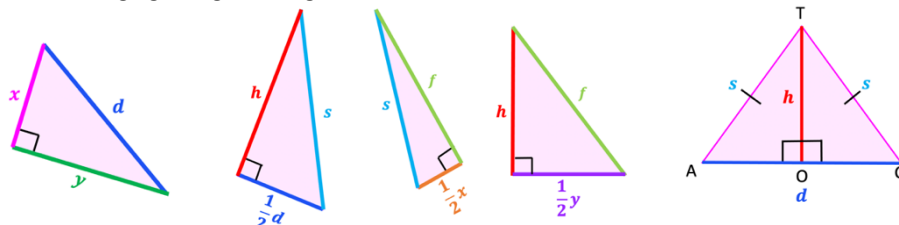


Pyramids

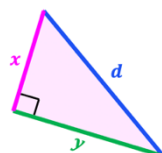
Being good at pyramids is simply just a question of being able to form the correct right-angled triangle. Consider the following pyramid



We commonly look at the 5 following right-angled triangles

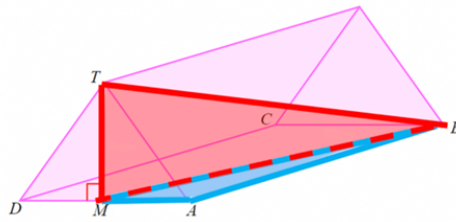


Take note how some of the lengths are half of the lengths of the width/lengths/diagonals of the base of the pyramid. It is key to realise this. The triangle that we normally look at first is

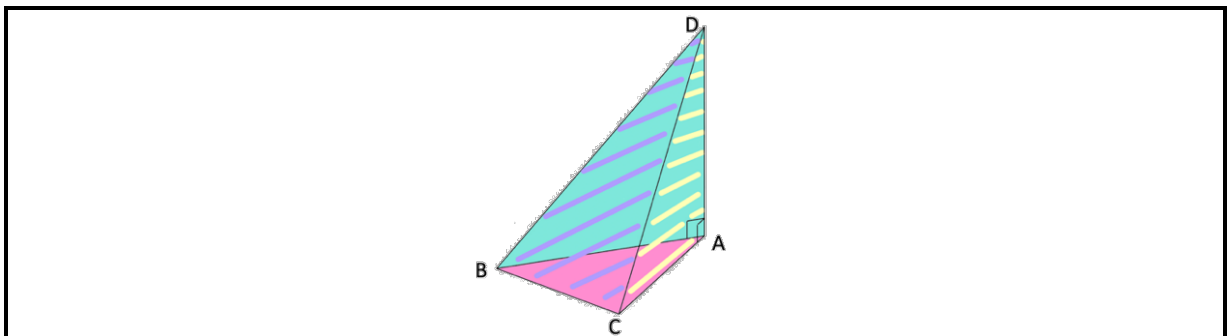


Triangular Prisms

We commonly look at the following 2 right-angled triangles (red and blue). We usually use Pythagoras or SOHCAHTOA on triangle MAB and Pythagoras or SOHCAHTOA on triangle TMB.



Tetrahedrons



We look at the following 4 triangles:

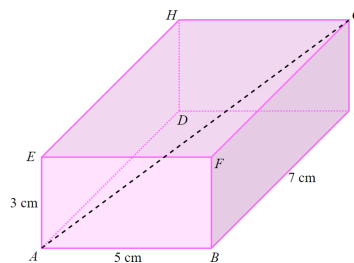
Right angle triangle – use Pythagoras or SOHCAHTOA		Non-right angle triangle – use sine or cosine rule	
<u>Side triangle</u>	<u>Side triangle</u>	<u>Top triangle</u>	<u>Bottom triangle</u>

1 Bronze



1.1 Cubes/Cuboids

- 1) The diagram shows a cuboid ABCDEFGH

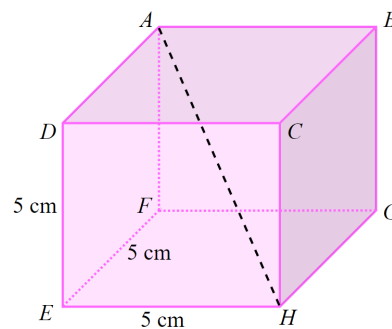


$$AB = 5 \text{ cm}$$

$$BC = 7 \text{ cm}$$

$$AE = 3 \text{ cm}$$

- i. Calculate the length AG
 - ii. Calculate the size of the angle between AG and the plane ABCD
- 2) The diagram shows a cuboid ABCDEFGH. Find
- i. the length AG
 - ii. the angle between AG and the plane ABCD
- 3) The diagram shows a cuboid ABCDEFGH. Find
- i. the angle between FA and the plane ABCD
 - ii. the angle between HE and the plane ABCD
 - iii. the angle between BH and the plane ABCD
- 4) The diagram shows a cube ABCDEFGH. The sides of the cube are length 5 cm. Calculate the angle between the diagonal AH and the base EFGH



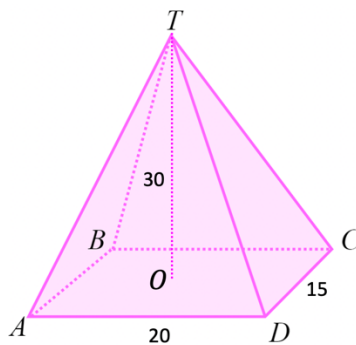
- i. Find the length of BH (7.07)
- ii. Find the length of AH (8.66)
- iii. Calculate the size of the angle between the diagonal AH and the base EFGH

2 Silver



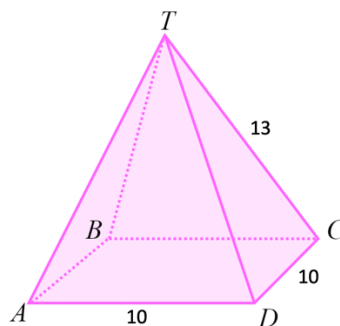
2.1 Pyramids

5) The diagram shows a pyramid



- Work out the slant edge TD
- Work out the size of angle TDO
- Find the size of the angle between TC and the base $ABCD$

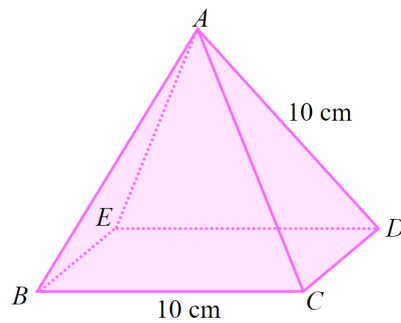
6) The diagram shows a pyramid



M is the midpoint of CD . Work out

- the length TM
- The vertical height
- The angle between TM and the base $ABCD$

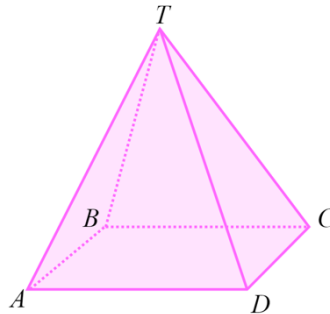
7) The diagram shows a pyramid.



BCDE is a square with sides of length 10 cm. The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

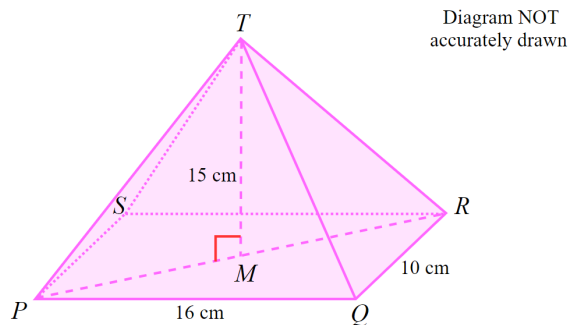
- i. Calculate the volume of the pyramid
- ii. Find the size of angle DAB

- 8) Here is a pyramid with a square base ABCD.



AB = 5 cm. The vertex T is 12 m vertically above the midpoint of AC. Calculate the size of angle TAC.

- 9) The diagram shows a pyramid with a horizontal rectangular base PQRS.



PQ = 16 cm

QR = 10 cm

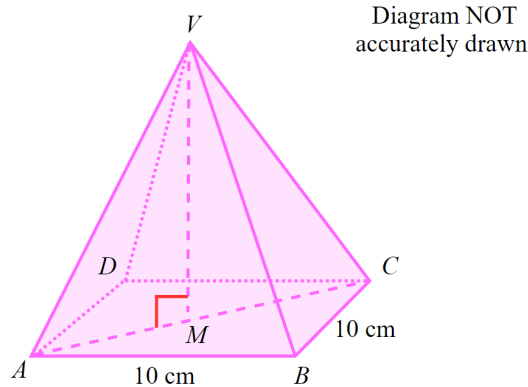
M is the midpoint of the line PR

The vertex T, is vertically above M

MT = 15 cm

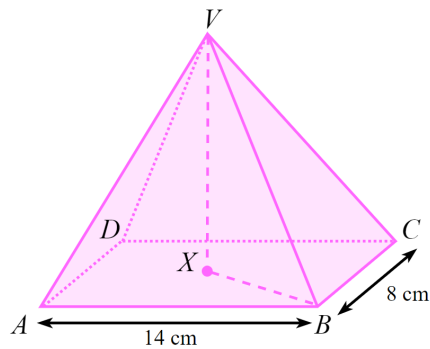
Calculate the size of the angle between TP and the base PQRS

- 10) The diagram shows a pyramid



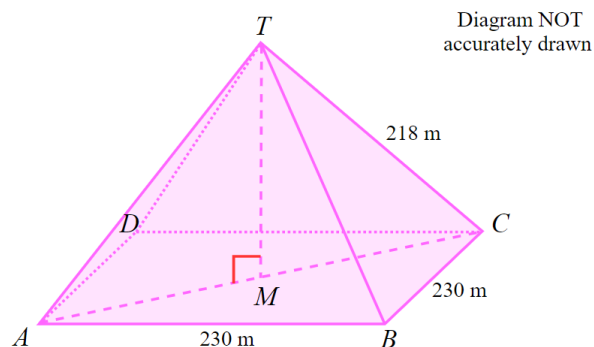
The base, ABCD, is a horizontal square of 10 side cm. The vertex, V, is vertically above the midpoint, M, of the base
 $VM = 12$ cm
 Calculate the size of angle VAM

- 11) VABCD is a rectangular based pyramid with volume 336 m^3 . X is the centre of the horizontal base, directly above V



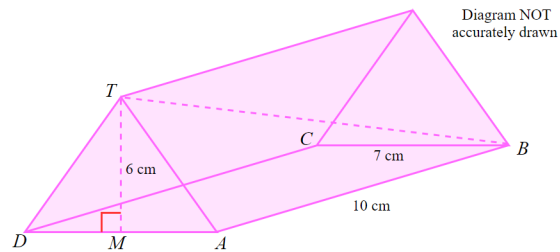
Work out the angle between VB and the base

- 12) A pyramid has a horizontal square base ABCD with sides of length 230 metres. M is the midpoint of AC. The vertex, T, is vertically above M. The slant edges of the pyramid are of length 218 metres. Calculate the height, MT, of the pyramid.



2.2 Triangular Prisms

- 13) The diagram shows a triangular prism with a horizontal rectangular base ABCD.



AB = 10 cm, BC = 7 cm.

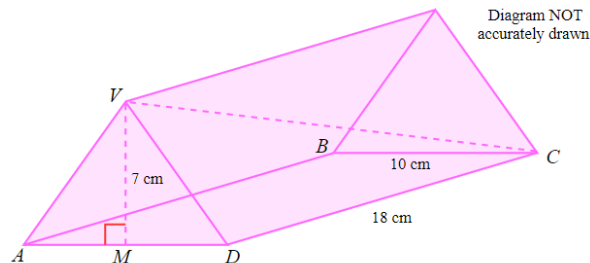
M is the midpoint of AD

The vertex T is vertically above M

MT = 6 cm

Calculate the size of the angle between TB and the base ABCD

- 14) The diagram shows a triangular prism with a horizontal base ABCD.



M is the midpoint of AD

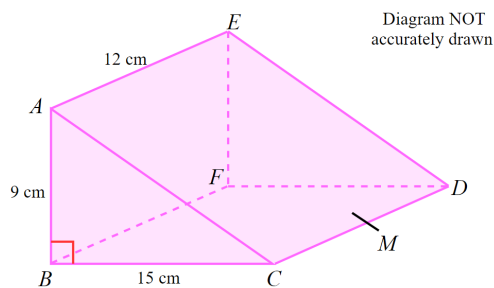
The vertex V is vertically above M

DC = 18 cm, BC = 10 cm, MV = 7 cm

Calculate the size of the angle between VC and the plane ABCD

2.3 Wedges

- 15) ACBDEF is a triangular prism



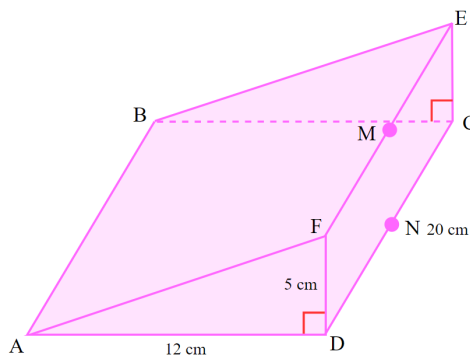
AB = 9 cm, BC = 15 cm and AE = 12 cm

Angle ABC = 90°

M is the midpoint of CD

Calculate the size of the angle between AM and the plane BCDF

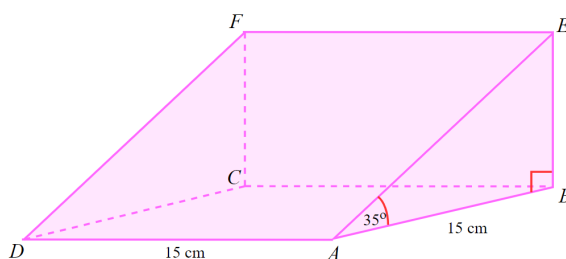
- 16) In the diagram below, ABEF, ABCD and CDFE are all rectangles.
 $AD=12$ cm, $DC=20$ cm and $DF=5$ cm
 M is the midpoint of EF and N is the midpoint of CD



Calculate

- i. The length of AF
- ii. The length of AM

- 17) The diagram shows a triangular prism



The base $ABCD$ of the prism is a square of side length 15 cm

Angle ABE and angle CBE are right angles

M is the point on DA such that $DM:MA=2:3$

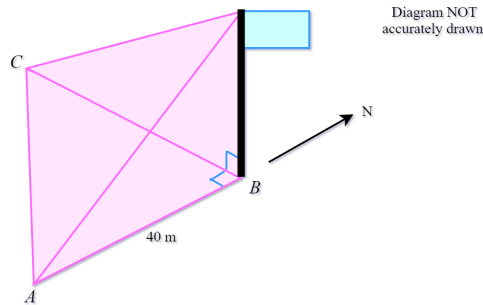
Calculate the size of the angle between EM and the base of the prism

3 Gold



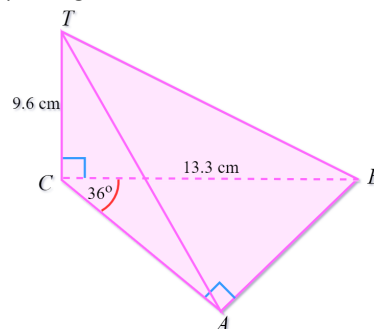
3.1 Tetrahedrons

- 18) A, B and C are points on a horizontal ground.
 C is due north of B
 A is due South of B and $AB = 40$ m
 There is a vertical flagpole at B
 From A, the angle of elevation of the top of the flagpole is 13°
 From C, the angle of elevation of the top of the flagpole is 19°



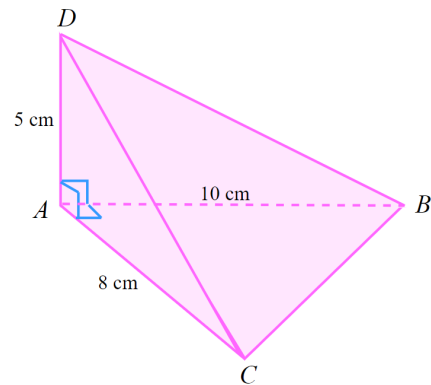
Calculate the distance AC

- 19) This 3D diagram represents a paperweight



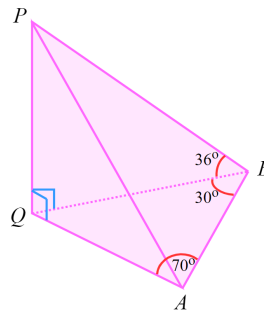
The horizontal base ABC is a right-angled triangle
 CT is vertical
 Angle $ACB = 36^\circ$, $BC = 13.3$ cm and $CT = 9.6$ cm
 Work out the size of the angle between AT and the horizontal base

- 20) The diagram shows a tetrahedron.



AD is perpendicular to both AB and AC.
 $AB=10\text{cm}$, $AC=8\text{cm}$, $AD=5\text{cm}$. Angle $BAC=90^\circ$
 Calculate the size of angle BDC

- 21) The diagram shows a vertical pole, PQ, which is supported by two wires fixed to the horizontal ground at A and B

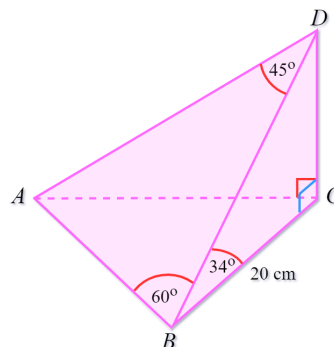


$BQ=40$
 $\angle PBQ = 36^\circ$
 $\angle BAQ = 70^\circ$
 $\angle ABQ = 30^\circ$

Find

- The height of the pole, PQ
- The distance between A and B

- 22) The diagram shows a pyramid with base ABC



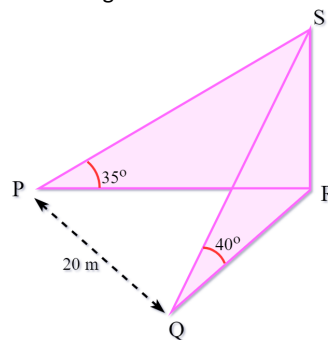
CD is perpendicular to both CA and CB
 Angle $CBD=34^\circ$, Angle $ADB=45^\circ$, Angle $DBA=60^\circ$,
 $BC=20\text{ cm}$
 Calculate the size of the angle between the line AD and the plane ABC

4 Diamond



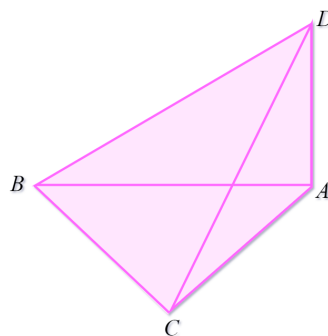
4.1 Harder Tetrahedrons

- 23) The three dimensional diagram shows the points P and q which are respectively **west and south-west** of the base R of a vertical flagpole RS on horizontal ground.



The angle of elevation of the top S of the flagpole from P and Q are respectively 35° and 40° and $PQ=20$ m. Determine the height of the flagpole.

- 24) The following three-dimensional diagram shows the four points A, B C and D. A, B and C are in the same horizontal plane and AD is vertical. $\hat{A}BC=45^\circ$, $BC=50$ m, $\hat{A}BD=30^\circ$, $\hat{A}CD=20^\circ$



Using the cosine rule in the triangle ABD, or otherwise find AD.

3D Trigonometry Questions By Topic Solutions:



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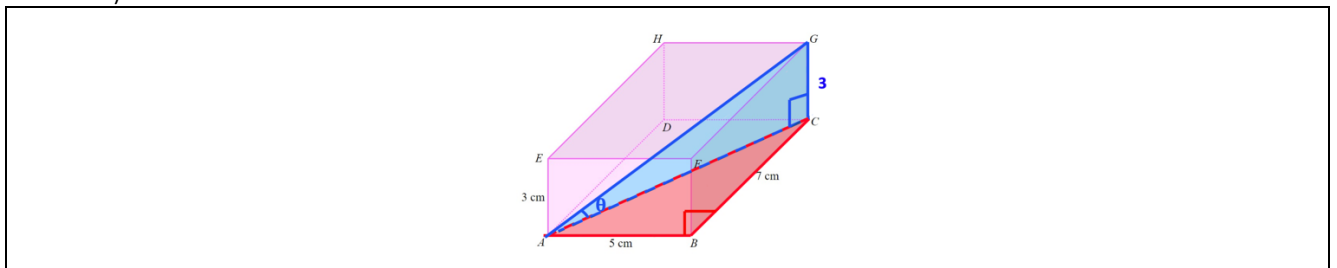
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1 Bronze



1.1 Cubes/Cuboids

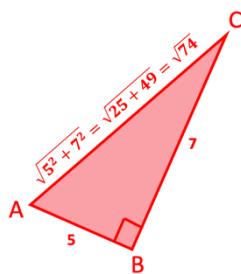
1)



i.

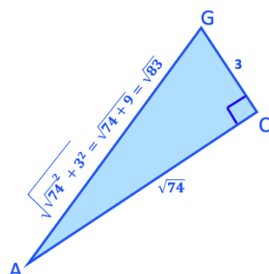
Step 1: Find AC using Pythagoras

$$AC = \sqrt{5^2 + 7^2} = \sqrt{74}$$



Step 2: Find AG using Pythagoras

$$AG = \sqrt{(\sqrt{74})^2 + 3^2} = \sqrt{74 + 9} = \sqrt{83} = 7.94$$

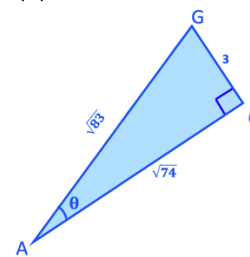


AG = 9.11 cm

Note: There is a shortcut to find the length AG. It is the square root of all 3 lengths squared and added together
 $\sqrt{3^2 + 5^2 + 7^2} = \sqrt{83} = 9.11$

ii.

Drop a perpendicular from line AG to plane ABCD. We drop it from the point on the line AG that doesn't always touch the plane (this is G since A already touches the plane ABCD) to the plane. This meets the plane ABCD at the point C. We then form the triangle with these points (A, C and G). The angle being asked for is the angle formed with the point on the line that was already touching the plane (A).



We have all lengths so can use either sin, cos or tan

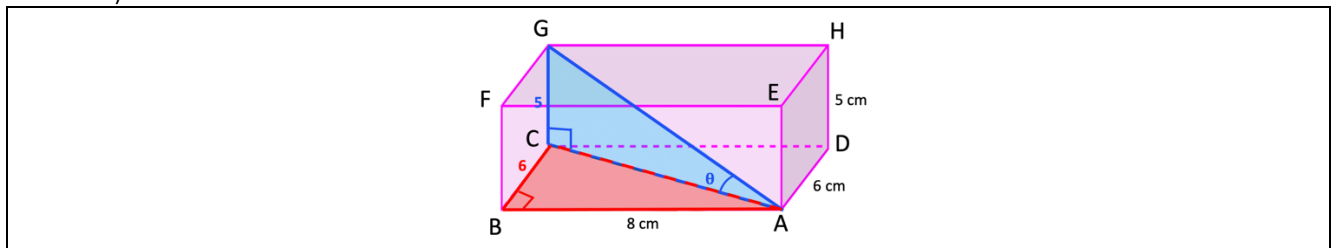
Let's use tan

$$\tan x = \frac{3}{\sqrt{74}}$$

$$\theta = \tan^{-1}\left(\frac{3}{\sqrt{74}}\right)$$

$$\theta = 19.2^\circ$$

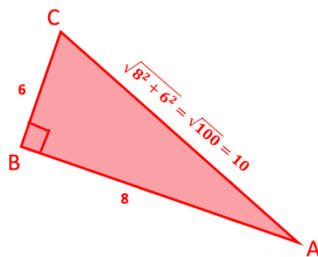
2)



i.

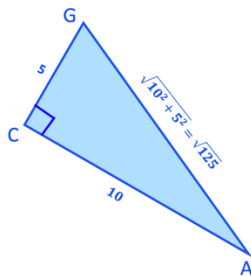
Step 1: Find AC using Pythagoras

$$AC = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$



Step 2: Find AG using Pythagoras

$$AG = \sqrt{10^2 + 5^2} = \sqrt{125} = 11.2$$



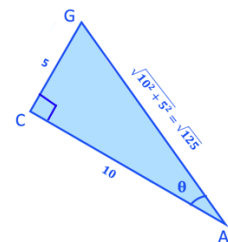
$$AG = 11.2 \text{ cm}$$

Note: There is a shortcut to find the length AG. It is the square root of all 3 lengths squared and added together

$$\sqrt{5^2 + 6^2 + 12^2} = \sqrt{205} = 9.11$$

ii.

Drop a perpendicular from line AG to plane ABCD. We drop it from the point on the line AG that doesn't always touch the plane (this is G since A already touches the plane ABCD) to the plane. This meets the plane ABCD at the point C. We then form the triangle with these points (A, C and G). The angle being asked for is the angle formed with the point on the line that was already touching the plane (A).



We have all lengths so can use either sin, cos or tan

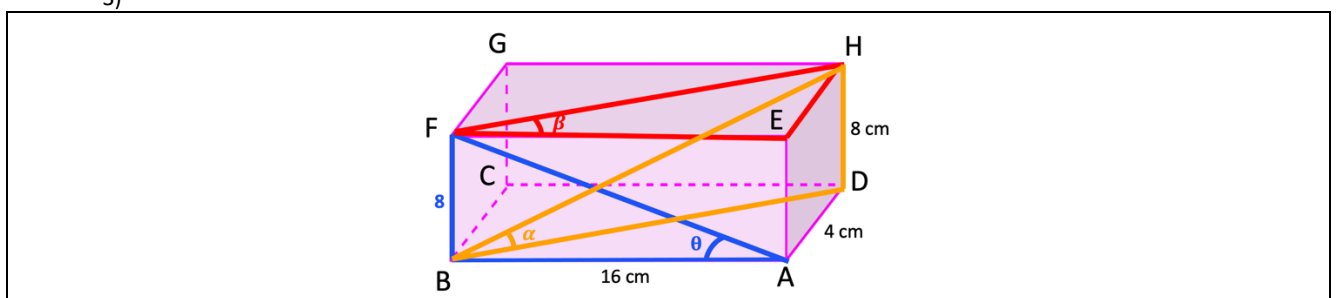
Let's use tan

$$\tan \theta = \frac{5}{10}$$

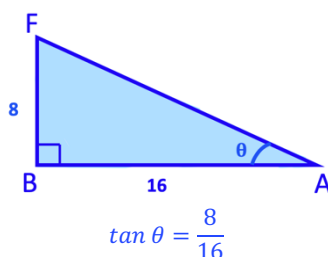
$$\theta = \tan^{-1}\left(\frac{5}{10}\right)$$

$$\theta = 26.6^\circ$$

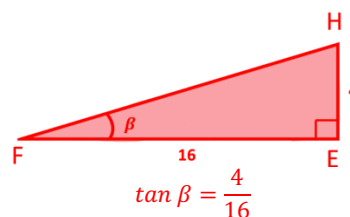
3)



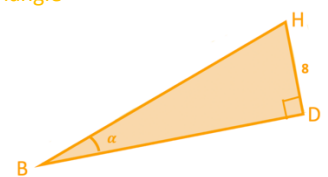
i. We need to form the following triangle



ii. We need to form the following triangle



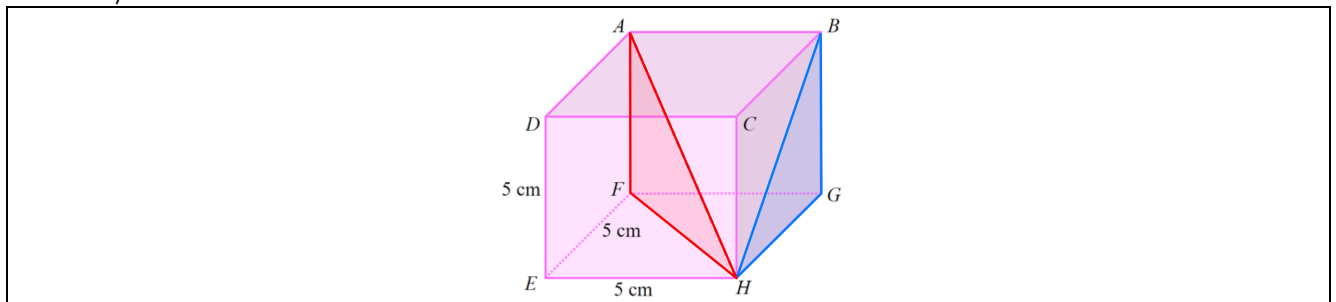
iii. We need to form the following triangle



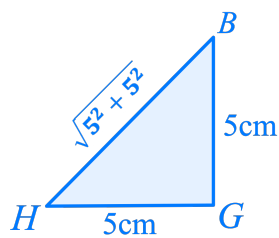
We need length BD or BH

$\theta = \tan^{-1}\left(\frac{8}{16}\right)$ $\theta = 26.6^\circ$	$\beta = \tan^{-1}\left(\frac{4}{16}\right)$ $\beta = 14.0$	<p>Let's Find BD by using Pythagoras on triangle BAD</p> $BD = \sqrt{16^2 + 4^2} = \sqrt{272} = 16.5$ $\tan \alpha = \frac{8}{16.5}$ $\alpha = \tan^{-1}\left(\frac{8}{16.5}\right)$ $\alpha = 25.9^\circ$
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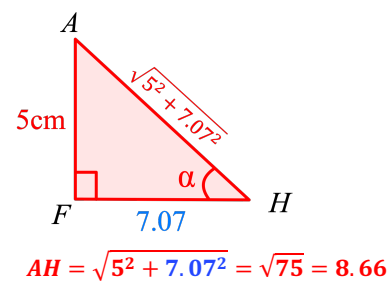
4)



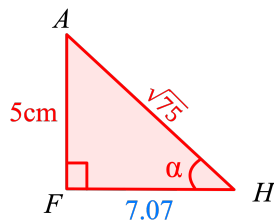
i. Find BH using Pythagoras
 $AC = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.07$



ii. Find AH using Pythagoras



i.



We have all lengths so can use either sin, cos or tan
 Let's use tan

$$\tan \alpha = \frac{5}{7.07}$$

$$\alpha = \tan^{-1}\left(\frac{5}{7.07}\right)$$

$$\alpha = 35.3^\circ$$

2 Silver



2.1 Pyramids

5)

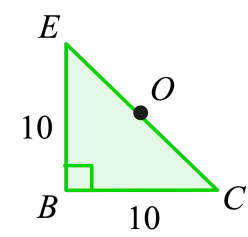
<p>We need to find BD first $BD^2 = 20^2 + 15^2$ $BD = \sqrt{625} = 25$</p> <p>$OD = \frac{1}{2}BD = y = \frac{1}{2}(25) = 12.5$</p> <p>Now we can find TC using Pythagoras $TD^2 = z^2 = 30^2 + 12.5^2$ $TD = z = \sqrt{1056.25} = 32.5$</p>	<p>We already found that $y = 12.5$ $\tan \alpha = \frac{30}{12.5}$ $\alpha = \tan^{-1}\left(\frac{30}{12.5}\right)$ $\alpha = 67.4^\circ$</p>	<p>We already found that $y = 12.5$ $\tan \alpha = \frac{30}{12.5}$ $\alpha = \tan^{-1}\left(\frac{30}{12.5}\right)$ $\alpha = 67.4^\circ$</p>

6)

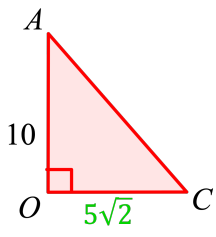
<p>i. It is best to use triangle TDM since triangle TOM has 2 many unknowns We can find TM using Pythagoras</p> <div style="text-align: center; margin: 10px 0;"> </div> $TM = y = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$	<p>i. Way 1:</p> <div style="text-align: center; margin: 10px 0;"> </div> <p>We know TM= 12 from part i. Using Pythagoras</p> $TTO = x = \sqrt{12^2 - 5^2} = \sqrt{119} = 10.9$ <p>Way 2:</p> <div style="text-align: center; margin: 10px 0;"> </div> <p>We can find OD by finding BD and halving it</p> $BD = \sqrt{10^2 + 10^2} = \sqrt{200}$ $OD = \frac{\sqrt{200}}{2} = 7.071$ $TO = x = \sqrt{13^2 - 7.071^2} = 10.9$	<p>ii.</p> <div style="text-align: center; margin: 10px 0;"> </div> <p>We know TM= 12 from part i.</p> $\cos \alpha = \frac{5}{12}$ $\alpha = \cos^{-1}\left(\frac{5}{12}\right)$ $\alpha = 65.4^\circ$

7)

a)	b)



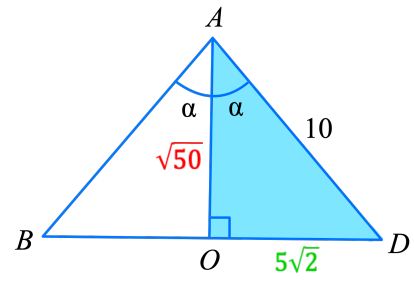
$EC = \sqrt{10^2 + 10^2} = 10\sqrt{2}$
 $OC = 5\sqrt{2}$



$EC = \sqrt{10^2 - (5\sqrt{2})^2} = \sqrt{50}$

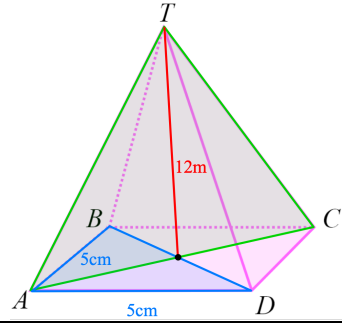
Therefore,

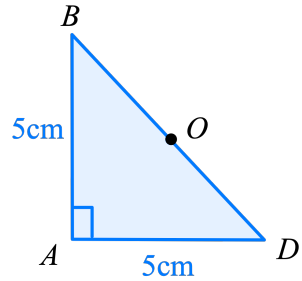
$\text{Volume} = \frac{1}{3}(10 \times 10)\sqrt{50}$
 $= 235.7 \text{ cm}^3$



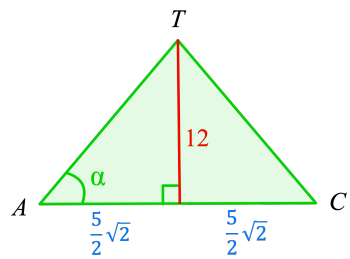
$\tan(\alpha) = \frac{5\sqrt{2}}{\sqrt{50}}$
 $= 45^\circ$
 $45(2) = 90^\circ$

8)



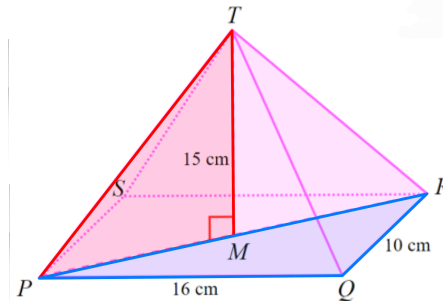


$EC = \sqrt{5^2 + 5^2} = 5\sqrt{2}$
 $BO = OD = \frac{5\sqrt{2}}{2} = AO$



$$\tan(\alpha) = \frac{12}{2.5\sqrt{2}} = 73.6^\circ$$

9) T

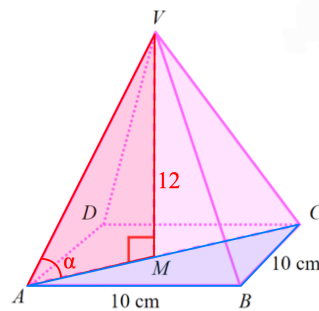


$$PR = \sqrt{16^2 + 10^2} = \sqrt{356}$$

$$PM = \frac{\sqrt{356}}{2} = 9.433$$

$$\tan(\alpha) = \frac{15}{9.433} = 57.8^\circ$$

10)

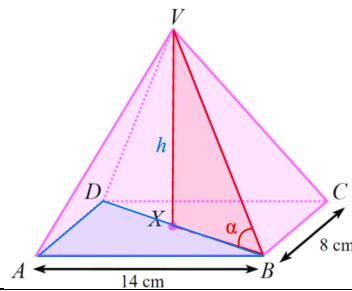


$$AC = \sqrt{10^2 + 10^2} = 10\sqrt{2}$$

$$AM = 5\sqrt{2}$$

$$\tan(\alpha) = \frac{12}{5\sqrt{2}} = 54.5^\circ$$

11)

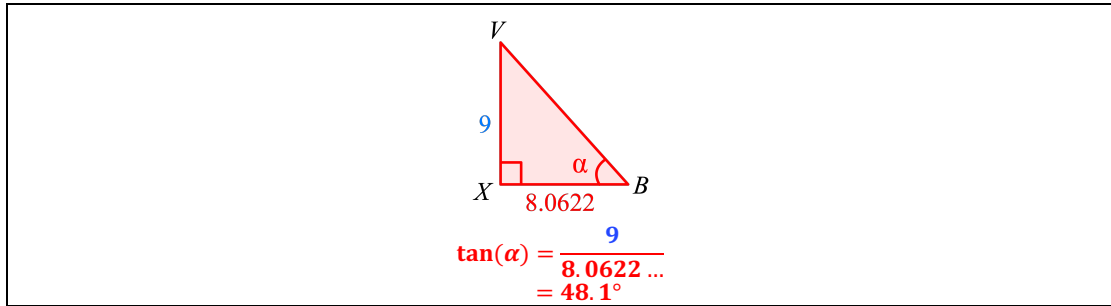


$$BD = \sqrt{14^2 + 8^2} = \sqrt{260}$$

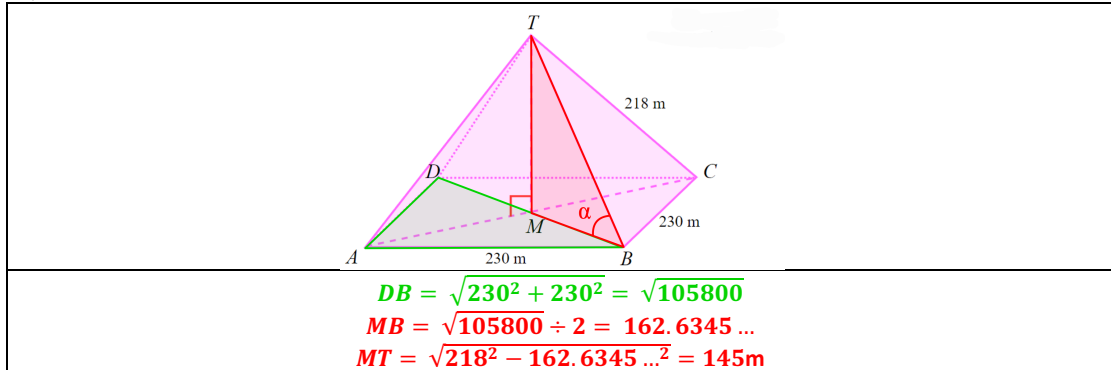
$$XB = \frac{\sqrt{260}}{2} = 8.0622 \dots$$

$$\text{Volume} = \frac{1}{3}(14 \times 8)h = 336$$

$$h = 9$$

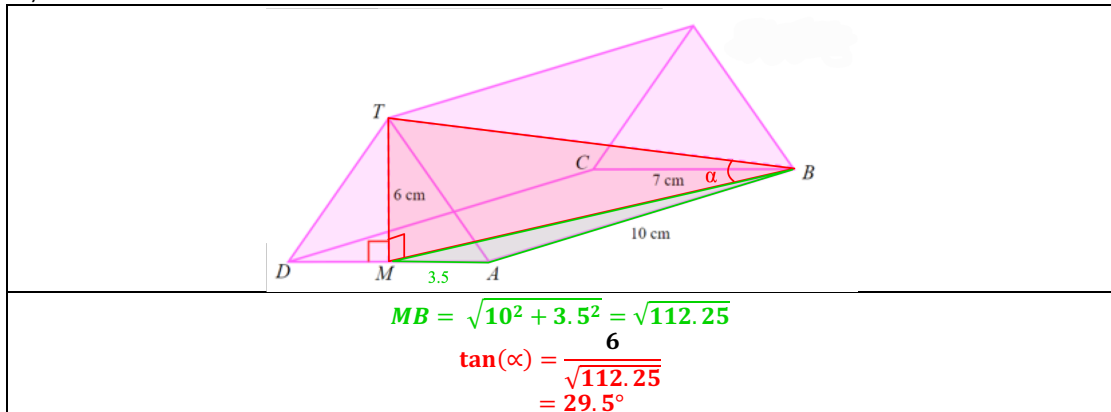


12)

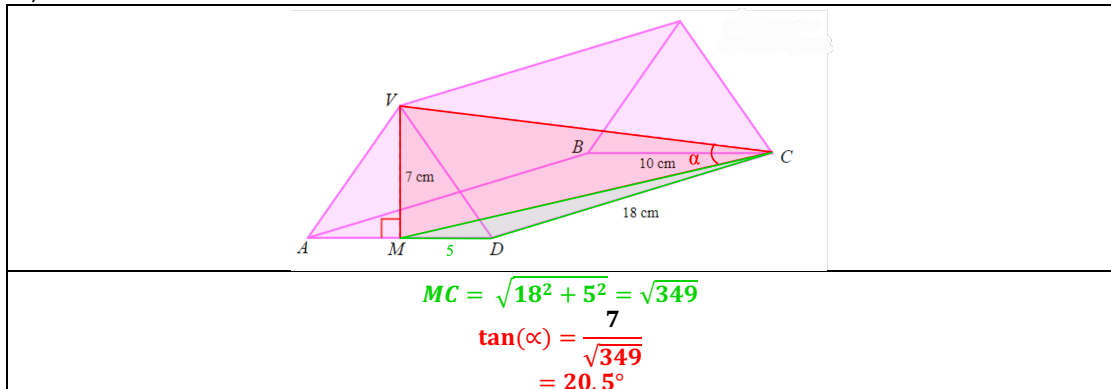


2.2 Triangular Prisms

13)



14)



2.3 Wedges

15)

$BM = \sqrt{15^2 + 6^2} = \sqrt{261}$
 $\tan(\alpha) = \frac{9}{\sqrt{261}}$
 $= 29.1^\circ$

16)

i.

$AM = \sqrt{12^2 + 5^2} = 13$

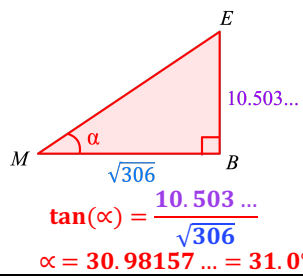
ii.

$AM = \sqrt{13^2 + 10^2} = 16.4$

17)

$x = \sqrt{15^2 + 9^2} = \sqrt{306}$

$\tan(35) = \frac{y}{15}$
 $y = \tan(35) \times 15$
 $= 10.50311307\text{cm}$



3 Gold



3.1 Tetrahedrons

18)

Look at the green triangle

$$\tan 13 = \frac{x}{40}$$

$$x = 40 \tan 13$$

$$x = 9.2346$$

$$\tan 19 = \frac{9.2346}{y}$$

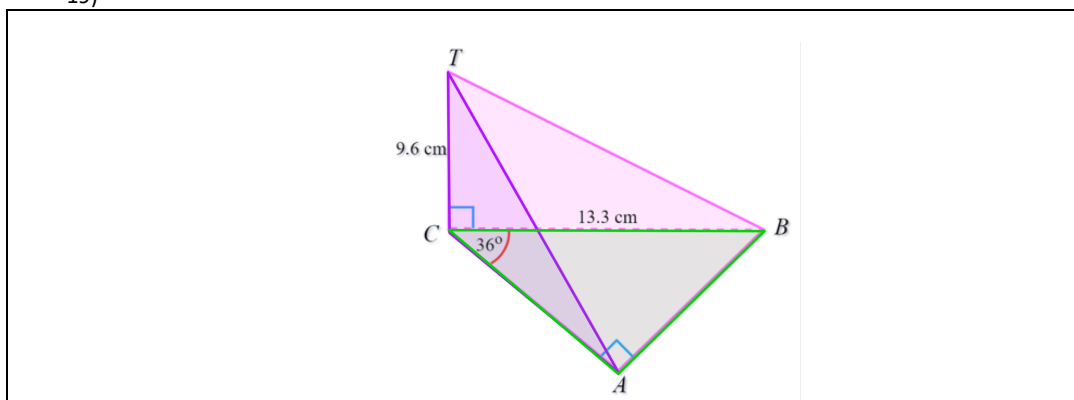
$$\tan 19 y = 9.2347$$

$$y = \frac{9.2347}{\tan 19} = 26.8195$$

Look at the pink base triangle and use Pythagoras

$$AC = \sqrt{40^2 + 26.8195^2} = 48.2 \text{ cm}$$

19)



Look at the green triangle

$$\cos 36 = \frac{x}{13.3}$$

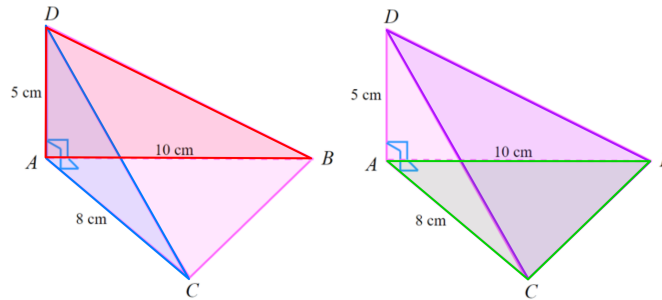
$$x = 10.759$$

Look at the purple triangle

$$\tan CAT = \frac{9.6}{10.759}$$

$$CAT = \tan^{-1} \left(\frac{9.6}{10.759} \right) = 41.7^\circ$$

20)



Look at the blue triangle and use Pythagoras

$$DC = \sqrt{5^2 + 8^2} = \sqrt{89}$$

Look at the red triangle and use Pythagoras

$$DC = \sqrt{5^2 + 10^2} = \sqrt{125}$$

Look at the green triangle and use Pythagoras

$$CB^2 = \sqrt{8^2 + 10^2} = \sqrt{164}$$

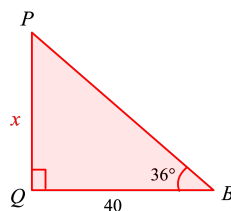
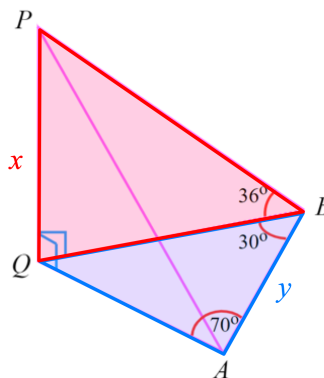
Look at the purple triangle and use cosine rule

$$164 = 89 + 125 - 2\sqrt{89}\sqrt{125} \cos BDC$$

$$\cos BDC = 0.237022$$

$$\text{Angle } BDC = 76.3^\circ$$

21)



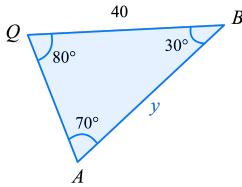
i.

Look at the red triangle

$$\tan 36 = \frac{x}{40}$$

$$x = 29.1\text{m}$$

ii.

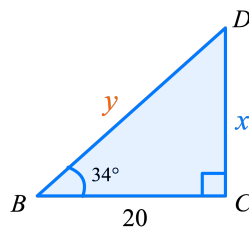
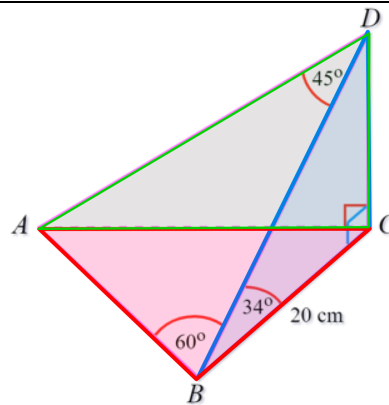


Look at the blue triangle

$$\frac{y}{\sin 86} = \frac{40}{\sin 70}$$

$$y = 41.9\text{m}$$

22)



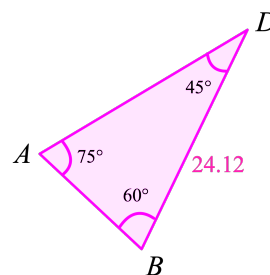
$$\tan 34 = \frac{x}{20}$$

$$x = 13.49$$

$$x^2 + 20^2 = y^2$$

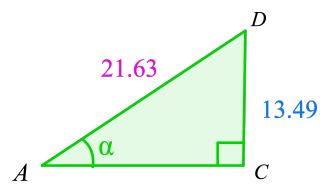
$$13.49^2 + 20^2 = y^2$$

$$y = 24.12$$



$$\frac{x}{\sin 60} = \frac{24.12}{\sin 75}$$

$$x = 21.63$$



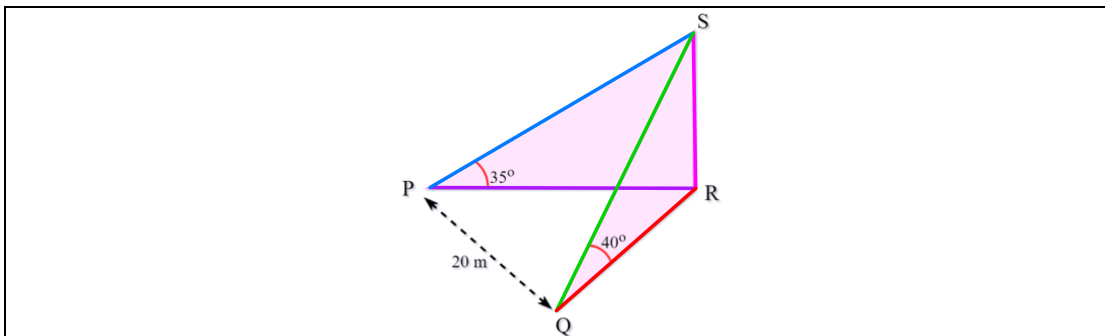
$$\sin \alpha = \frac{13.49}{21.63}$$
$$\alpha = 38.6^\circ$$

4 Diamond

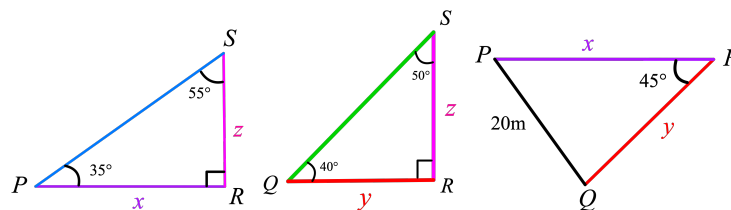


4.1 Harder Tetrahedrons

23)

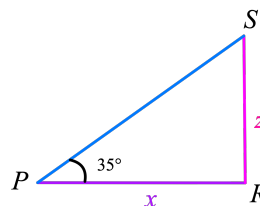


We will look at the 3 following triangles out of the possible 4
 Right angled triangle PRS
 Right angled triangle QRS
 Non-right angled triangle PQR
 We don't need to look at triangle PSQ



Let's get the sides x and y in terms of z since z is common to both triangles

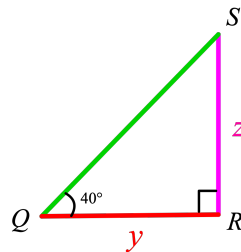
Looking at right angled triangle PRS:



$$\tan 35 = \frac{z}{x}$$

$$x = \frac{z}{\tan 35}$$

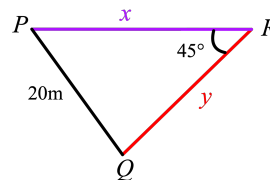
Looking at right angled triangle QRS:



$$\tan 40 = \frac{z}{y}$$

$$y = \frac{z}{\tan 40}$$

Looking at non right-angled triangle PQR:



This is a non right angled triangle so we must use cosine rule. We have both x and y in terms of z , so we can plug these into the cosine rule

$$20^2 = x^2 + y^2 - 2(x)(y) \cos 45$$

$$20^2 = \left(\frac{z}{\tan 35}\right)^2 + \left(\frac{z}{\tan 40}\right)^2 - 2\left(\frac{z}{\tan 35}\right)\left(\frac{z}{\tan 40}\right) \cos 45$$

$$400 = \left(\frac{z}{\tan 35}\right)^2 + \left(\frac{z}{\tan 40}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{z}{\tan 35}\right)\left(\frac{z}{\tan 40}\right)$$

$$400 = \frac{z^2}{(\tan 35)^2} + \frac{z^2}{(\tan 40)^2} - \left(\frac{2}{\sqrt{2}}\right)\left(\frac{z^2}{\tan 35 \times \tan 40}\right)$$

$$400 = z^2 \left(\frac{1}{(\tan 35)^2} + \frac{1}{(\tan 40)^2} - \sqrt{2} \left(\frac{1}{\tan 35 \times \tan 40} \right) \right)$$

$$z^2 = \frac{400}{\left(\frac{1}{(\tan 35)^2} + \frac{1}{(\tan 40)^2} - \sqrt{2} \left(\frac{1}{\tan 35 \times \tan 40} \right) \right)}$$

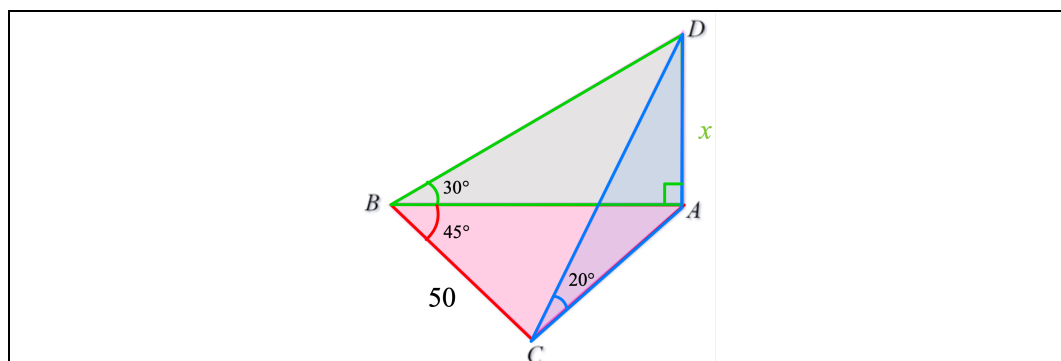
$$z = \sqrt{\frac{400}{\left(\frac{1}{(\tan 35)^2} + \frac{1}{(\tan 40)^2} - \left(\frac{2}{\sqrt{2}}\right) \left(\frac{1}{\tan 35 \times \tan 40} \right) \right)}}$$

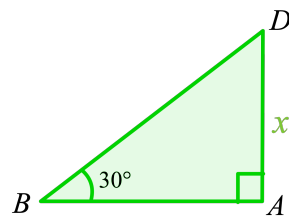
$$z^2 = \frac{400}{2.0396 + 1.420 - 2.407}$$

$$z = \sqrt{\frac{400}{2.0396 + 1.420 - 2.407}} = 380.011$$

$$z = 19.5 \text{ m}$$

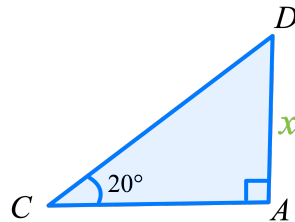
24)





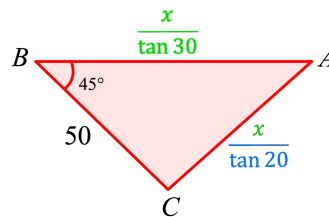
$$\tan 30 = \frac{x}{AB}$$

$$AB = \frac{x}{\tan 30}$$



$$\tan 20 = \frac{x}{AC}$$

$$AC = \frac{x}{\tan 20}$$



Now we use the cosine rule

$$AC^2 = BC^2 + AB^2 - 2(BC)(AB) \cos 45$$

$$\left(\frac{x}{\tan 20}\right)^2 = (50)^2 + \left(\frac{x}{\tan 30}\right)^2 - 2(50)\left(\frac{x}{\tan 30}\right) \cos 45$$

$$\frac{x^2}{(\tan 20)^2} = 2500 + \frac{x^2}{(\tan 30)^2} - 100\left(\frac{1}{\sqrt{2}}\right)\left(\frac{x}{\tan 30}\right)$$

$$\frac{x^2}{(\tan 20)^2} - \frac{x^2}{(\tan 30)^2} + 100\left(\frac{1}{\sqrt{2}}\right)\left(\frac{x}{\tan 30}\right) = 2500$$

$$7.549x^2 - 3x^2 + 122.474x - 2500 = 0$$

$$4.549x^2 + 122.474x - 2500 = 0$$

Use the quadratic formula to solve this

$$x = 13.6, -40.5$$

x can't be negative since it is a side length

$$x = 13.6$$